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B

Length of a Day

INTRODUCTION

In this teaching unit, we want students to measure or calculate:

- The time of sunrise and sunset on a given day,
- the length of that day, and
- the graphical representation of the heights of the sun on the horizon throughout the day. Students can also keep tabs with the data obtained for one day, then repeat the calculations for another day and compare the two.

Students of this unit should be between 15 and 18 years old because they need prior knowledge of trigonometry and astronomy.

N.B.: For the purpose of analysing the length of days according to seasons, "seasons" are those of the Northern hemisphere.

Some notes on astronomy

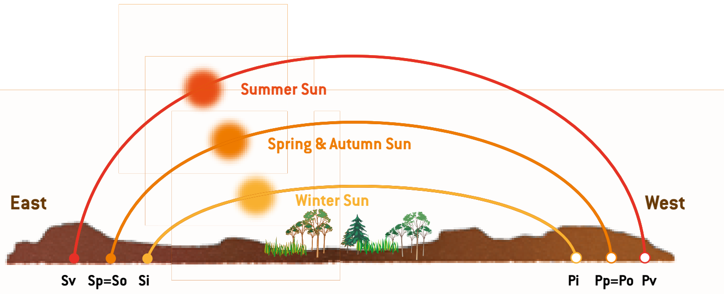
The sun's daily path across the sky varies throughout the year. In the summer, the sun is at its highest point in the sky. In the winter it follows a lower path, which is why the days are longer in summer than in winter. In the spring and in autumn the sun describes intermediate paths, as shown in the next figure. ①

On the first day of spring, the sun crosses the celestial equator (the declination of the sun is 0). During subsequent days, the sun's apparent motion follows higher orbits until the first day of summer when it reaches the maximum (declination ε). The next day, the trajectory is lower in the sky and goes down until the first day of autumn when it returns to tour the equator (declination 0), continuing down to the first day of winter at the lowest point (declination -ε). The sun rises every day to get back to the first day of spring that runs along the equator, thus restarting the cycle of a new year.

Length of a day describes the moment from when the upper limb of the sun disc appears above the horizon during sunrise, to the moment when the upper limb disappears below the horizon during the sunset.

The length of day varies throughout the year and depends on the latitude. The inclination of the Earth's rotation axis causes the seasons to change and the position of sunrise and sunset to change every day. The maximum angular distance between two sunrises or two sunsets is the angle between two solstices. This angle changes with the latitude of the place. It is at its minimum along the equator

① Path of the sun on the first day of each season

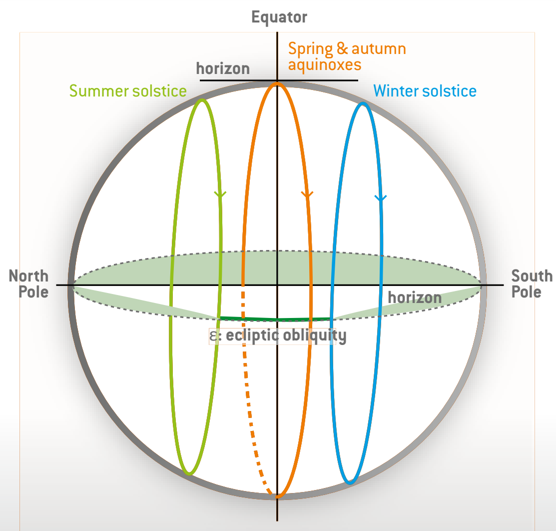


Sv, Sp, So, Si dots indicate the sunrise in summer, spring, autumn, & winter.
Pv, Pp, Po,Pi dots indicate the setting sun in summer, spring, autumn & winter.

(where it is equal to ecliptic obliquity ε). After that it increases according to the absolute value of the latitude, until it causes the midnight sun in the polar area. So, in an equatorial city (latitude $\phi = 0°$), the distance between two sunsets can be, at maximum 2ε (between June and December solstices), see ②. In any place along the equator, the length of day and night is always the same: 12 hours.

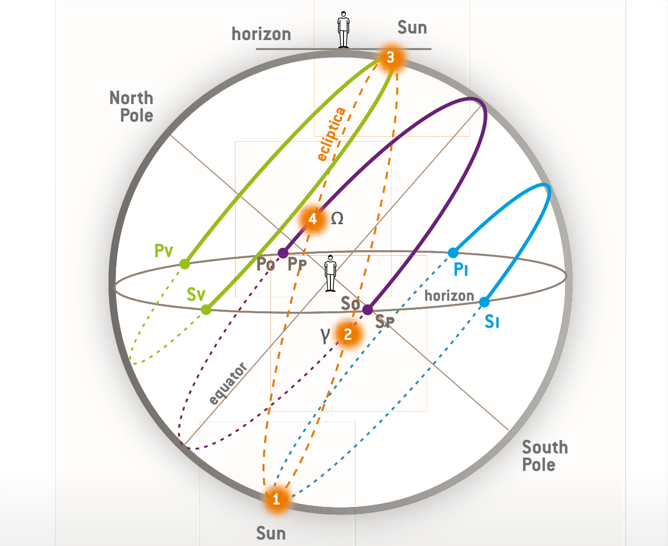
At the Pole the sun's path is parallel to the horizon (midnight sun) and it is not possible to consider the angular distance between two sunsets because we don't have sunset points. In places near the polar circle, the length of day (or night) can vary from one day to six months.

② The sun's path at latitude 0° (on the equator)



Our city Zaragoza has a latitude above 40° N. It is for this zone that we calculate the length of day and its variation for different times of year. In our region, day and night are of equal duration during two days of the year: the days of the equinoxes. From spring equinox to the autumnal equinox, days are longer than nights. Finally, from the autumnal equinox to the spring equinox, the length of the night exceeds length of the day. In ③ we present the path of the sun, the days of the solstices, and equinoxes for latitudes similar to ours.

③ The sun over our horizon



In the blue the winter solstice, in purple the equinoxes and in green the summer solstice

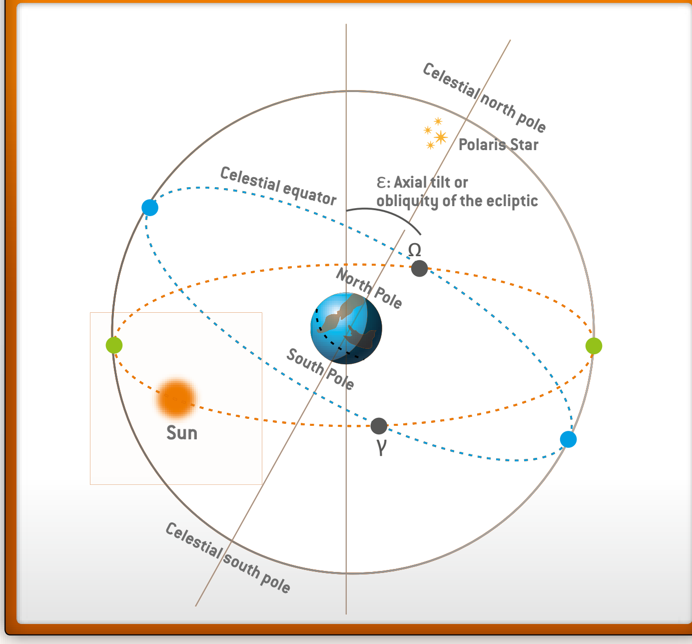
But, what are the ecliptic and the obliquity of the ecliptic?

The ecliptic is the plane of the Earth's orbit around the sun. In other words, it is the intersection of the celestial sphere with the plane containing the Earth's orbit around the sun (ecliptic plane).

As the axis of rotation of the Earth is not perpendicular to the ecliptic plane, the equatorial plane is not parallel to that ecliptic plane. An axis perpendicular to the ecliptic and the Earth's rotation axis creates an angle of about 23°26', and is called the obliquity of the ecliptic. It is represented by ε.

The intersections of the equatorial and ecliptic planes with the celestial sphere generate two maximum circles, known as the celestial equator and the ecliptic respectively. The line of intersection between these two planes at two opposite points leads to the equinoxes (see ④).

④ Ecliptic and equinoxes



The Vernal equinox (or point of Aries) occurs when the sun passes from south to north. The autumn equinox (or point of Libra), occurs when the sun passes from north to south. The obliquity of the ecliptic is not a fixed quantity but changes over time in a cycle of about 26,000 years. The slow and gradual change in the orientation of the axis of rotation of the Earth is due to the torque exerted by the tidal forces of the moon and sun on the Earth's equatorial protuberance. These forces tend to carry excess mass present in Equator to the plane of the ecliptic.

RESOURCES

For the first part: (introduction and presentation of the work) we used a Mac OS X computer, version: 10.4.11. and Word and Adobe Illustrator CS for figures.

For the development of the application we used Eclipse IDE with Java, on a Windows system.

To check the calculated values in the Java application, it would be desirable to have a Pin Hole camera or a stick, a string and a protractor, so that students can perform the measurements themselves, using simple tools.

CORE

The Java program (see www.science-on-stage.de) for calculating the length of day is divided into two sections. The left-hand side is for introducing some parameters such as the date, the latitude and longitude of a place. This left-hand side will also show the numerical results for the time of sunset, sunrise and the value of the length. The right-hand side will show the sun's highest point of the day for the location requested. The line starts at the sunrise date and time, evolves into the higher value, and decreases until the time of sunset.

There are also three buttons ‚Calculate', ‚Clear Values' and ‚Clear Sun Paths' that will allow you to reset given values, to start the calculation, and to clean the sun's path graph.

Calculations registered by the program can be found on the teaching-unit version on the Internet. They can also be used to calculate the length manually. However, as this is a complex process, we advise you to use the Java program to obtain different results and to complete the analysis.

Let us check, for example, how, in the same place, and over a period of one year, the heights evolve, when entering different values. The next figure shows the results. ⑤

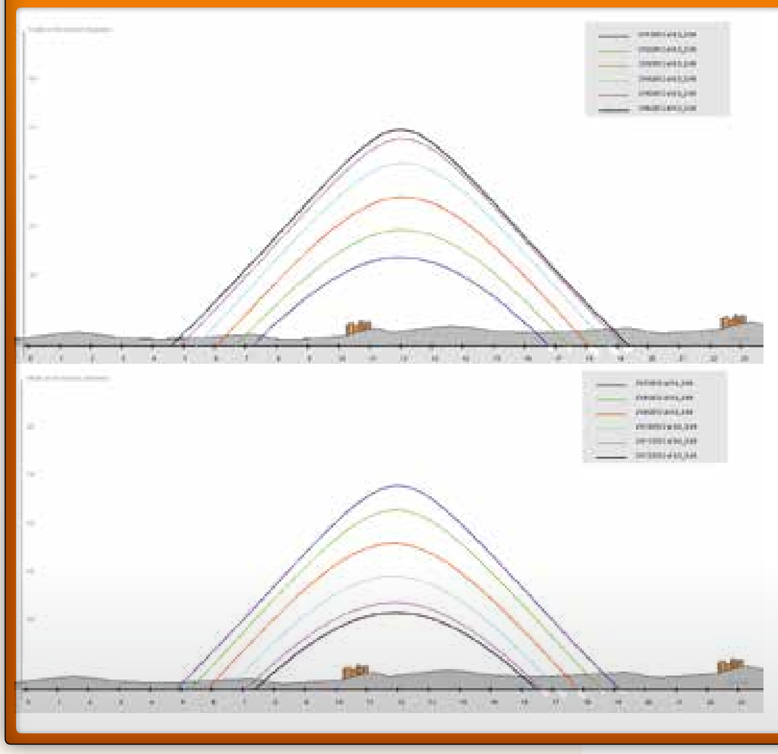
In the last figure we see how the height of the sun increases until the month of June and also how the length of day increases by having earlier hours of sunrise and later hours of sunset. However, from July to December the height starts to decrease, influencing both the length of the day and the time of sunrise and sunset.

Another interesting thing to check is how the height of the sun differs for different locations on the same day. For example, for 21st June 2012, see the difference between 40º North and 40º South of the equator. It is interesting to see that the time of sunset and sunrise is more or less the same but the height can vary by more than 60º between the equator and the North Pole.

Varying only the longitude but maintaining the date and the latitude could generate another analysis. The result should be that the length of day and the height reached are the same but the times of sunrise and sunset time differ from the longitude time we entered.

It is also interesting to see that a day can last 12 hours during the equinoxes (around 21st March and 21st September). The maximum length takes place during the summer solstice (around 21st June) and the minimum length occurs during the winter solstice (around 21st December).

⑤ **Comparison of the sun path of the same location in each month**



As a final proposal for students, it could be interesting to verify some of the results obtained by the Java program by creating their own simple device. For example, using a Pin Hole camera, they can reproduce the variation of the sun's height throughout a day.

Using a simple stick, students can also calculate the angle formed by sunbeams and the horizon. This angle is the angular altitude (height) of the sun at this moment. Students can check the results for different times of the day and find out that the values measured with this simple device are similar to the values they obtained with the Java program.

Another way for students to make these calculations could be to mark the points on the ground where the shadows of the top of the stick are found throughout a day.

CONCLUSION

The Java application that we developed works for any day of the year and any terrestrial latitude. But while using the program, students may come up with strange results. For certain latitudes, the sun does not rise and set on some days, so you cannot measure the length of day. The program then issues a red text, warning us, that we are in a place where people can enjoy the midnight sun in the summer. In contrast, some winter days are dark 24 hours.

The program can calculate the length of day for different dates and save the graphical representation for each of them. That way, we can compare the change in the time of sunrise and sunset, depending on the season, and as result find out the length of day.

As a special project for students, it could be interesting to assign calculations for different latitudes to different groups of three or four students each. Depending on the number of students, they can have designated latitude zones covering 15 or 20 degrees, for both the northern and southern hemisphere. Using their calculations, each group can prepare a graph in a PowerPoint presentation to show to fellow students and to discuss the results obtained within the different groups.

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