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C

Phases of the Moon

INTRODUCTION

Have you ever noticed that regardless of which place on Earth we are, we all see the same shape of moon on the same day? Have you ever noticed that the illuminated part of the moon changes sequentially and cyclically?

In this teaching unit, we'd like students to understand how the relative positions of the sun, earth and moon influence each phase of the moon, how to determine this phase for a given day, and how to calculate the percentage of its illuminated part.

This unit is recommended for students aged 14 to 16 years because some prior knowledge in trigonometry and astronomy is required.

Some notes on Astronomy

When we speak of the phase of the moon, we are referring to the lighted portion of the moon, as seen by an observer on Earth. This appearance changes cyclically, when the moon orbits the Earth, according to the relative positions of the earth, moon and sun to each other. One half of the lunar surface is always illuminated by the sun, but the portion of the illuminated hemisphere that is visible to an observer on Earth can vary from seeing the whole lunar disk illuminated (full moon) to not seeing it at all (new moon).

Early in time, it was recognised that the moon's shape depends upon its "age", that is, the number of days that have elapsed since the previous new moon. In ①, the inner circle shows the moon's orbit, assuming it is circular, the earth being at its centre. The sun's direction is indicated by the sunlight, and since the sun's distance is some 400 times the moon's, we can assume that the sun's direction, as seen from the moon, is always parallel to its geometric direction. The moon's illumination being provided by the sun, the day and night sides of the moon at different parts of its orbit will be as shown in ①.

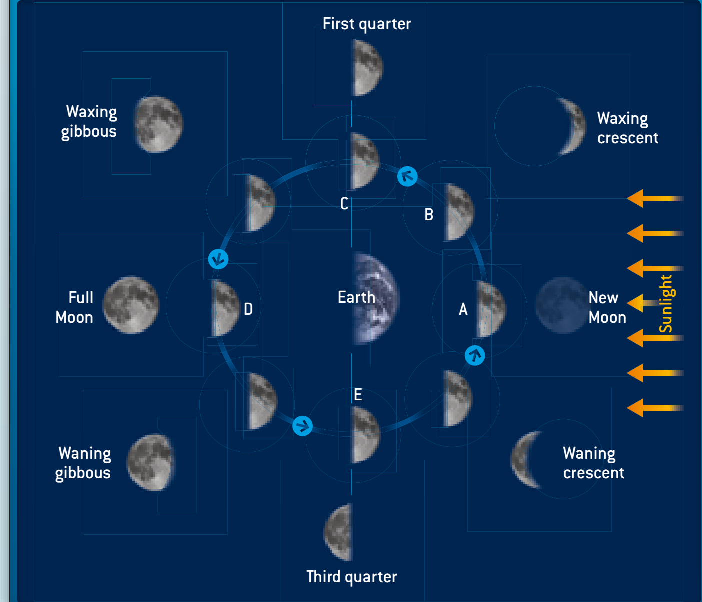
The outer circle of figures shows the appearance of the moon as seen from Earth, in other words: the phases of the moon. At point A the moon is new; at B we see a waxing crescent (waxing means growing and refers to the size of the illuminated part of the moon, which is increasing). First quarter occurs at C; between C and E, more than half the moon's illuminated face is visible, a condition known as gibbous. At D it is full moon; at E the position is called third quarter. Between E and A is a waning crescent (when the moon is said to be waning, we see a little less of the moon each day, until it completely disappears, at which point the moon is new).

We can now define the moon's synodic period, lunation or lunar month. Although the moon's orbit undergoes changes, an average value, defined to be the time interval between two successive new moons, has been determined for this period. This value, named Sc, is 29.53059 days.

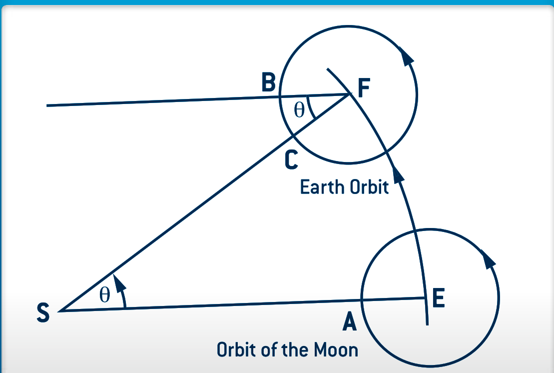
The moon's sidereal period or sidereal month is the time interval required by the moon to make one complete revolution around the Earth. With respect to the stellar background, this is the path from A to B in ②. Again, we can determine an average value. Here it is 27.32166 days.

The difference between these two periods of time is due to the fact that the moon has to travel a little further in its

① The day and night sides of the moon



② Relation of sun, moon and earth from a geometrical point of view



orbit to catch up with the sun which, from the geometric point of view, is also revolving around Earth (Earth has passed from E to F in ②, while the moon has to reach point C, and not point B, to be a new moon, as seen in point A). The three quantities, namely: the sidereal periods of revolution of the moon around the earth, the earth around the sun, and the moon's synodic period, must then be related to each other.

RESOURCES

First part: For the introduction and presentation of the work, we used a Mac OS X computer, version: 10.4.11. Applications: Word and Adobe Illustrator CS for figures.

For the development of the application, we used an Eclipse IDE (see annex) with Java 1.6 and the library Java3D. The application can be found at www.science-on-stage.de, where you can download it as well as download the source.

CORE

In this section, we are going to explain the steps needed to calculate the phase of the moon of a given day in the northern hemisphere. Students then could calculate the phase manually, or, if they prefer, use it as a base for programming an application like the Java version that we have prepared for ICT.

Input

The only data needed for calculating the phase of moon is the date for which the students want to know its phase. This date represents a day, a month and a year.

Analysis

1. First, the students start working with the given date (day, month, year). This date is transformed into Julian Days (JD is a system of time measurement used by the astronomy community. It presents the interval of time in days elapsed since the epoch 1900 January 12 a.m., because it is midnight at Greenwich on 31 December 1899). The hour is fixed at 12:00 a.m. So, given a {day, month, year} to calculate in Julian days, we need to solve the following simple equations:

$$a=\frac{(14-month)}{12}$$

$$y=year+4800-a$$

$$m=month+12*a-3$$

So, JD [day, month, year] of

$$JD[day, month, year]=day+\frac{(153\cdot m+2)}{5}+365\cdot y+\frac{y}{4}-\frac{y}{100}+\frac{y}{400}-32045$$

is the day chosen in Julian's Day format.

2. A reference date of a past new moon is also needed, for example 1st January of 1900. This date needs to be transformed into a Julian Day just like in the previous step.

Remember that if JD $[1,1,1900]_{Reference}$ is the reference date, it is not possible to calculate moon phases before that date.

3. The next step is to calculate the difference between the date we want and the reference date:

$$JD[x]_{Current}-JD[x]_{Reference}=D$$

This calculation allows you to find out how many days passed since this known new moon.

4. As we explained, Sc is the time interval between two successive new moons. With that, if we made an integer division D/Sc, the rest will be the days passed since the last new moon. Calling the rest A, A will be the age of the moon. So, Age of the moon = A = D modulus Sc

5. As Sc is 29.53059, and the rest of the division is zero, the phase of the moon will be new. Thus, the rest can take on values between 1 and 29, with 29 being the equivalent to zero or new moon.

Now it's easy to assign a number to each value of the rest of the phases. We do this in a counter-clockwise direction, see Figure 1. So, a value of 0 is equivalent to new moon, a value of 7.38 corresponds to the first quarter, 14.76 stands for a full moon and 22.15 represents the third quarter.

6. If, in addition to viewing the phase of the moon on our chosen day, we want to calculate the percentage of the illuminated part, we have to use the formula

$$Percentage=\frac{1}{2}\left(1-\cos\left(\frac{360}{S_c}\right)*A\right),$$

which gives us just that percentage. If P = 0, the phase is new moon, if P = 1, it is a full moon. But if P = ½, is it the first or third quarter?

Here we need to take some other aspects into consideration. Let's call A the age of the moon used on the previous formula, and η = 360*(A/Sc). The η is called the moon's elongation. See figure ②B. When the sun, the earth and the moon are aligned in the order cited $\eta = 180^\circ$ and there is a full moon, and 29/2 days have passed since the last new moon, we can take a look at Figure ②B and make the following assumptions:

If $0 < A \leq 29/2 \rightarrow 0 < \eta \leq \pi$ then we have two cases:

- For $0 < \eta < \pi/2$ the moon is waxing crescent, the shadow is on the left and the illuminated part is less than half the lunar disk ③
- For $\pi/2 < \eta < \pi$ the moon is waxing gibbous, the shadow is on the left and the illuminated portion is greater than half the lunar disk ④

If $A = 29/2 \rightarrow \eta = \pi \rightarrow$ full moon.

If $A \geq 29/2 \rightarrow \pi < \eta \leq 2\pi$ then we have two cases:

- For $\pi < \eta < 3\pi/2$ the moon is waning gibbous, the shadow is on the right and the illuminated portion is greater than half the lunar disk ⑤.
- For $3\pi/2 < \eta < 2\pi$ the moon is waning crescent, the shadow is on the right and the illuminated part is less than half the lunar disk ⑥.

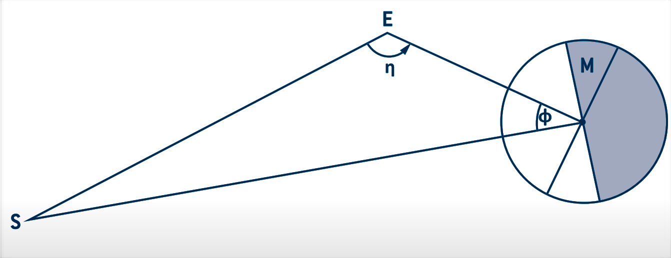
That said, we are able to specify: If P=½, the moon is in first or third quarter. Similarly, we can deduce, for example: If a percentage value of 0.8 corresponds to the right or the left side of the lunar disk, then the phase is a waxing or waning crescent, respectively.

Output

Having completed the analysis, the students are able to know which phase corresponds to the given date and what percentage of the moon's surface is illuminated.

As part of the unit, a Java application is developed. Students and teachers could use it to better understand the influence of the relative positions of the sun, earth and moon to each other, and during certain phases of the moon, or they can verify their results.

②B The moon's elongation



In this application, there are three zones: an info panel of the current phase at the left side, an animation with the sun, earth and moon at the right side, and the text fields for entering the date at the bottom.

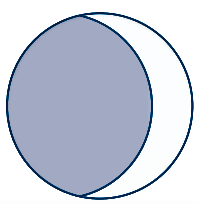
In the animation panel there are two buttons to see the animation, 'Play' and 'Stop'. Use these to control the positions of the moon, earth and sun. Depending on the position, the information panel on the left side will show you the current phase of the moon.

For calculating a certain phase for a date, students only have to enter the day, month and year in the bottom text fields and press 'Calculate'. The info panel and the animation will update their content showing information about the calculated moon phase.

If they want to calculate the phase manually, they only have to follow the previously explained steps, and verify their results with the application.

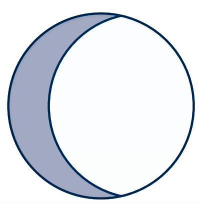
As said before, this program can calculate the phase of the moon for any day in the northern hemisphere. We encourage students to investigate how the inhabitants of the southern hemisphere see the moon on a given day. Do they see the same phase we do? How does the view of a phase differ (other than the new and full moon) from one hemisphere to another? Can you explain that differ-

③ Waxing crescent phase



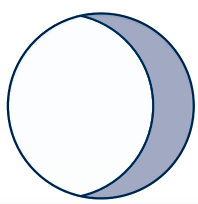
$0 < A < 29/2$ $0 < \eta < \pi/2$

④ Waxing gibbous phase



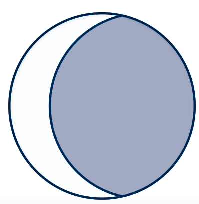
$0 < A < 29/2$ $\pi/2 < \eta < \pi$

⑤ Waning gibbous phase



$A > 29/2$ $\pi < \eta < 3\pi/2$

⑥ Waning crescent phase



$A > 29/2$ $3\pi/2 < \eta < 2\pi$

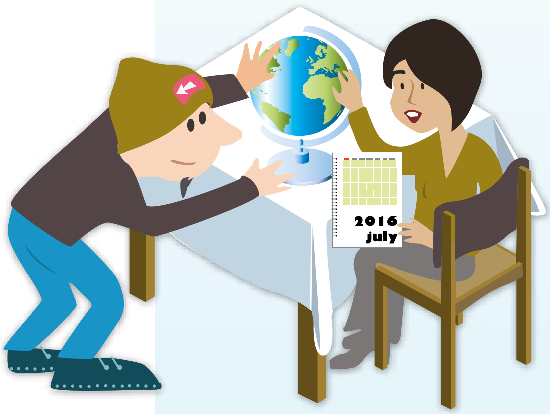
ence? And finally, we encourage students to make a program that allows them to visualise the phases in the southern hemisphere.

CONCLUSION

This unit presents a guided method to calculate the phase of the moon for a given date.

Teachers are advised to encourage their students to learn these basic concepts of astronomy as well as follow the simple steps for calculating and explaining moon phases.

Both teachers and students could also use the Java application to better understand the process, check their results, or just compare phases for consecutive days. The Java source code is also a good way to program these kinds of simulations.



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