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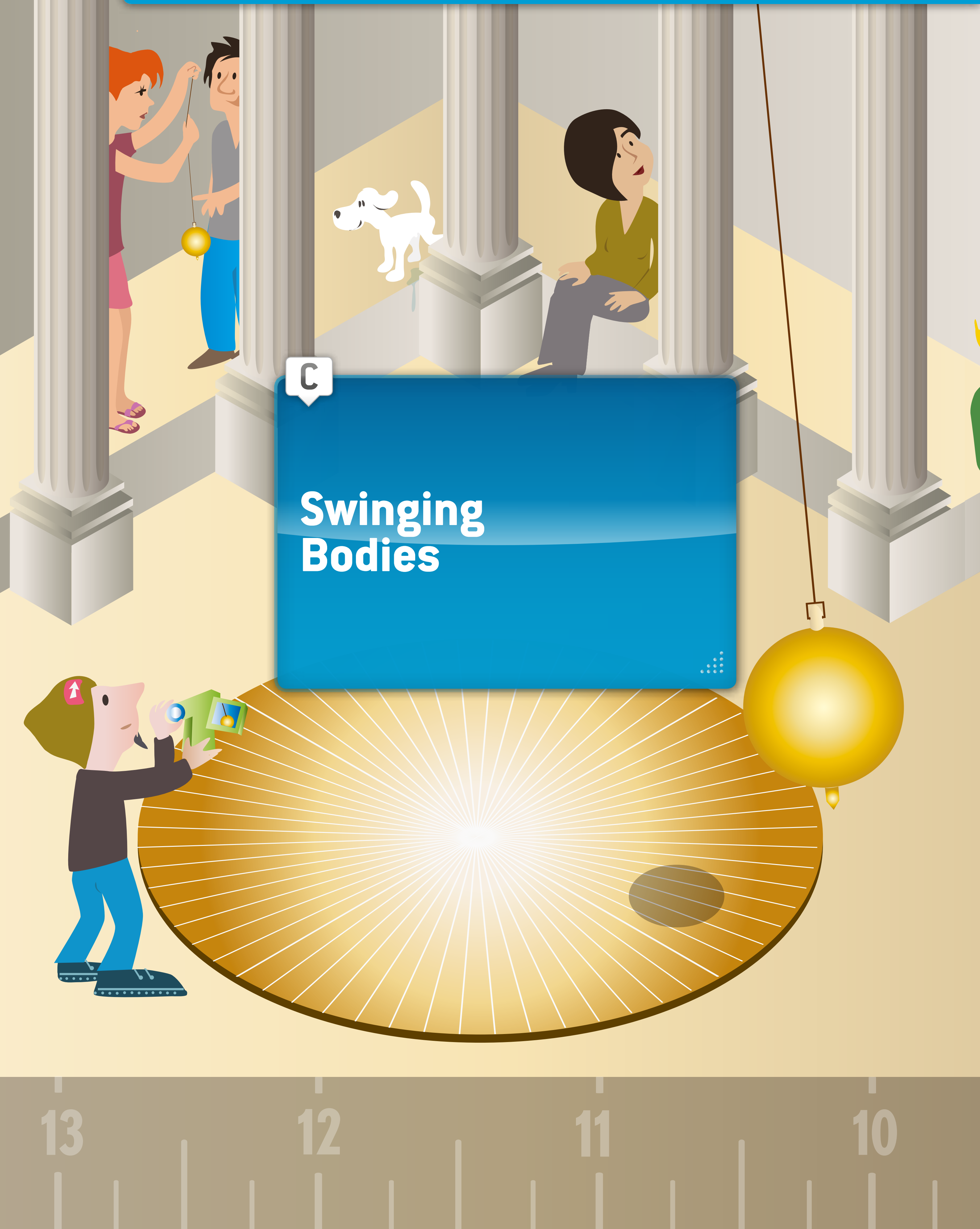
C

Swinging Bodies



INTRODUCTION

Oscillating objects are all around us. Each and every sound is produced by an oscillating source. Studying oscillation is not the easiest thing to do but we've broken it down to a simple concept of spring and pendulum motions.

The unit is recommended for students aged 14 to 16 years (level I), and for students aged 17 to 19 years (level II). The applied subjects are: physics, mathematics and information and communication technologies (ICT).

Level I

Students set up the spring or pendulum and make it oscillate. They observe the simple motion and record the oscillations with a video camera or mobile phone camera. Using Tracker or VirtualDub (see annex), they analyse the videos (frame by frame) to specify the motion character (displacement-time dependence). With the help of these videos and the graphical analysis, the students are able to determine frequency, time period, amplitude, and the spring constant or acceleration of gravity for a pendulum.

Level II

- **A:** These students take the same steps as younger students but they analyse the graph in greater detail. With the help of these videos and the graphical analysis, students can observe the phase shifts for displacement, and are able to find the following quantities: frequency, time period, amplitude, velocity, acceleration, and their dependence upon time. They also can verify the Law of Conservation of Mechanical Energy.
- **B:** The students have to add an accelerometer to the body of oscillation. They can register the acceleration values and applying this data, they can calculate period, velocity, amplitude, displacement, kinetic and potential energies. Then they plot graphs and may verify the parameters of the same motion using these two methods: differentiation (displacement → velocity → acceleration) and integration (acceleration → velocity → displacement).

RESOURCES

To work with this unit, students need: a digital video camera, a web cam or a mobile phone camera; a ruler, or other type of scale (to be placed in close proximity of the oscillating body and visible on the video); different kinds of springs, 3 to 4 objects of different weight, hanging from a spring; 3 to 4 pendulums of different lengths, a computer or a laptop; video-analysis software, e.g.: Tracker or VirtualDub; the Java application "Osc" available at www.science-on-stage.de.

CORE

The simplest mechanical oscillating systems consist of a body of mass m hanging from a spring or a pendulum (small oscillating angle). The inertia property of the mass m causes the system to overshoot the point of equilibrium. By applying Newton's second law to the oscillating body, one can obtain the equation of motion for the system.

The students have to revise the formulae for different physical quantities.

Level I

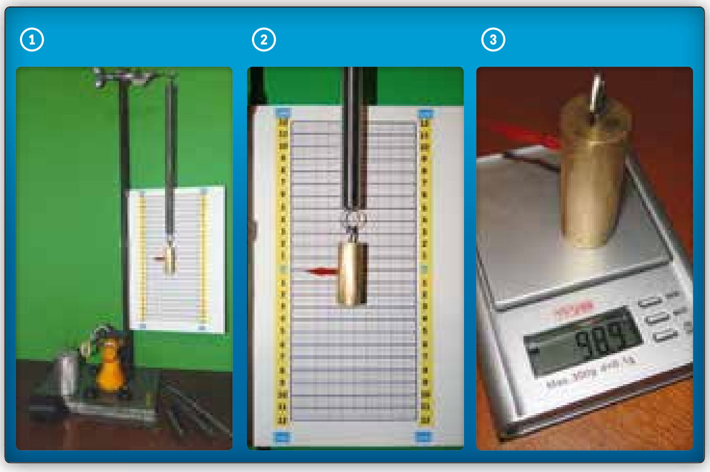
Level I students should revise the following physical quantities:

- The spring period: $T = 2\pi\sqrt{\frac{m}{k}}$, where m is the mass of the oscillating body
- The pendulum period $T = 2\pi\sqrt{\frac{\ell}{g}}$, where ℓ is the length of the pendulum and g is the gravitational acceleration

Level II

Level II students should revise the following physical quantities:

- Elastic force: $F = kx$, where k is the spring constant, x is the displacement of the oscillating body
- The time periods: for a spring $T = 2\pi\sqrt{\frac{m}{k}}$, where m is the mass of the oscillating body; for a pendulum $T = 2\pi\sqrt{\frac{\ell}{g}}$, where ℓ is the length of the pendulum and g is the gravitational acceleration
- Displacement of the oscillating body in simple harmonic motion: $x = A \sin(\omega + \varphi)$, where A is the amplitude, ω is the angular frequency, and ϕ is the phase constant
- Displacement of the oscillating body in a damped oscillation: $x = Ae^{-(b/2m)t}\cos(\omega t+\Phi)$ with $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$, where b is the viscous damping coefficient
- Velocity of the oscillating body $v = \omega A \cos(\omega t + \varphi)]$
- Acceleration of the oscillating body $a = -\omega^2 A \sin(\omega t + \varphi)$



- Total mechanical energy can be written as the sum of potential and kinetic energy:

  for a spring $E_m = E_p + E_k = \frac{ky^2}{2} + \frac{mv^2}{2}$

  for a pendulum $E_m = E_p + E_k = mg\Delta h + \frac{mv^2}{2}$

Experiment for Level I and II

- Set up the spring with a body of mass hanging from the bottom of the spring / set up the pendulum with a body of mass hanging from the bottom; place the ruler in the proper position to measure the displacement ① ②
- Record the mass (spring experiment)/record the length (pendulum experiment) ③
- Set up the computer webcam facing the spring/pendulum, so that it is able to capture the whole set
- Drop the body of mass from the original position and let it oscillate towards its mean position
- Save the video
- Measure the time period with a chronometer, or read it from the recorded movie
- Add an accelerometer to the body of oscillation and save the data (only level II)
- By changing selected parameters, find out how they affect the oscillation quantities

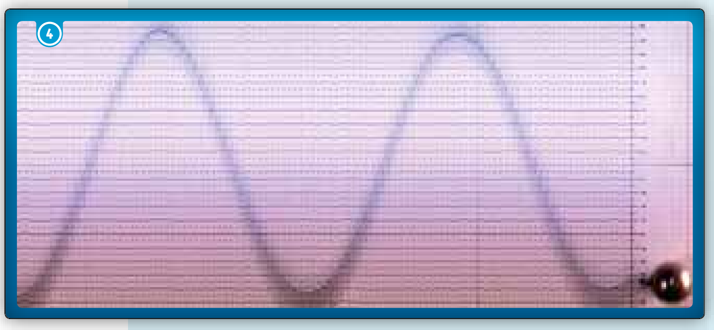
Analysis

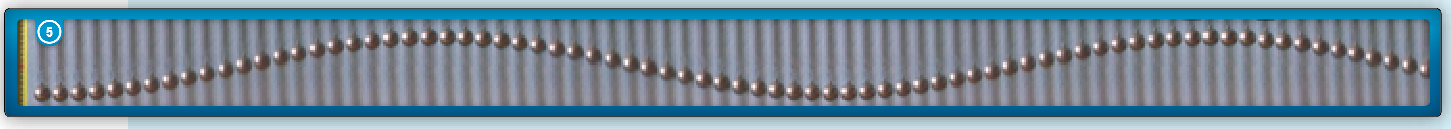
1. To start working with Tracker software, the students have to import the video clip and choose parts to be analysed.

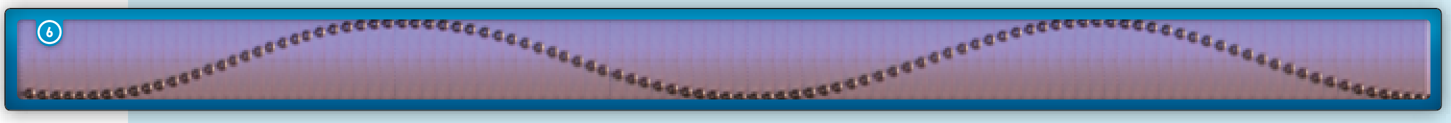
The program processes information about the position of the observed body as a function of time. Based on this data, the program plots graphs with the time dependence of different quantities: position in the horizontal and vertical dimensions, speed in these two dimensions, actual velocity, acceleration, mechanical energy (kinetic and potential). If students want to observe and analyse their variations, the program also allows you to define some new physical quantities.

2. Working with Tracker or VirtualDub, students may observe the common character of displacement variations for both spring and pendulum oscillations. Figures 4-7 are the combination of time-lapse images made with VirtualDub software. Comparing these images, one recognises similar characteristics between pendulum and spring oscillations.

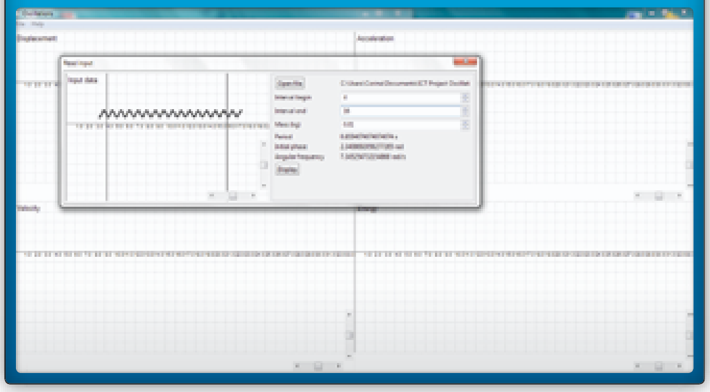
- Spring oscillations (summarised frame by frame) ⑤
- Pendulum oscillations (summarised frame by frame) ④
- Pendulum (summarised frame by frame) ⑥



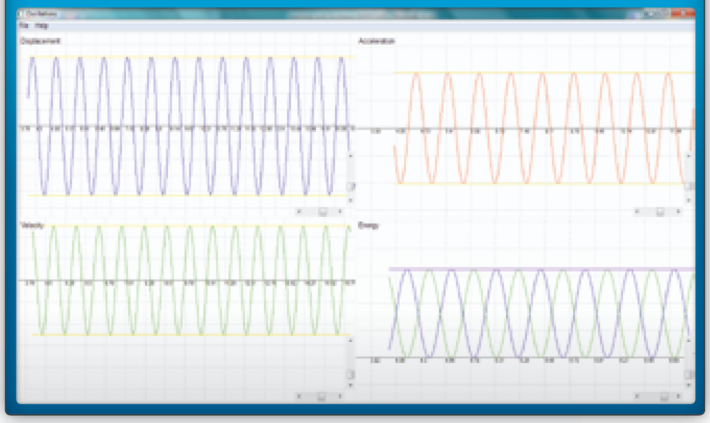




⑦ **Comparison of the experimental data and the simulation data**



⑧ **Diagram made with the software Osc**



3. An interesting way to study a simple harmonic motion of a spring/pendulum is to use an accelerometer, and to record the acceleration of the oscillating body. Then the students can process this data using the software "Osc" available at www.science-on-stage.de. The software provides four graphs with the time dependence of acceleration, velocity, displacement and total energy (kinetic and potential).

If students import the data, they have to draw the graph: a = f(t). From this graph, they can estimate the time period of the motion and calculate the angular frequency and displacement of the oscillating body. They then compare the experimental data with the data provided by the software. ⑦

Questions for Summarisation

Using Tracker, Virtual Dub or Osc students may work on the following main tasks:

- Observe the character of the oscillations (level I, II)
- Find out the characteristics for oscillations (I, II)
- Plot the graphs: T = f(m), if k is constant and T = f(k), if the mass is constant (level II for spring), and T = f(l) (level I, II for pendulum)
- Observe as the phase shifts between displacement and velocity and between displacement and acceleration (II)
- Verify the law of Conservation of Mechanical Energy – on the graph ⑧; the black curve represents the total energy, which equals the sum of the potential energy (blue curve) and the kinetic energy (green curve) (II)
- Check that the time period of the potential and kinetic energy variation is half the oscillation-time period (II)
- Verify the dependence T = f(m) for a spring with the constant k, if there are corresponding files with data for different masses, or the dependence T = f(k) for the same body of mass and different springs (II)
- Verify the dependence T = f(l) for a pendulum (I, II)

Using the same software "Osc" at www.science-on-stage.de, students can simulate a damped oscillation ⑨. They may choose the oscillation's parameters: frequency, amplitude, phase constant and also b/2m (where b is the viscous damping coefficient and m is the mass of the oscillating body) (II). The students may formulate their opinion about: displacement values at the moment when velocity or acceleration reach their maximum or the point of 0, the difference between the motion period and kinetic or potential energy periods, and, last but not least, the influence of friction on the parameters of the motion.

⑨ **Simulation of a damped oscillation with the software Osc**

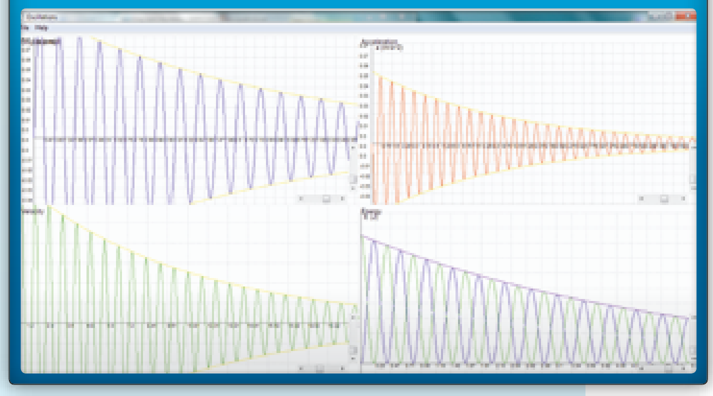


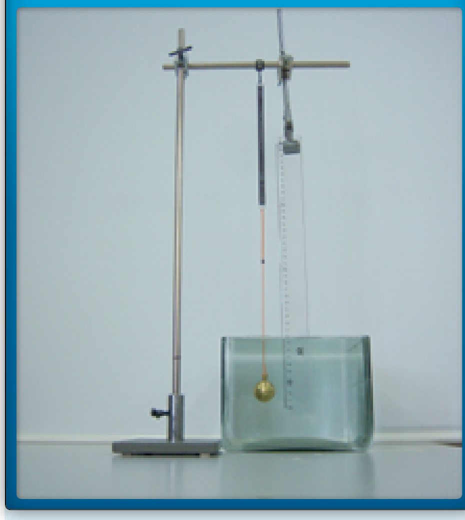
Figure ⑩ shows how to perform a simple set for testing the damped oscillations. Figure ⑪ is the result of analysis made with Tracker.

The students may formulate conclusions about:

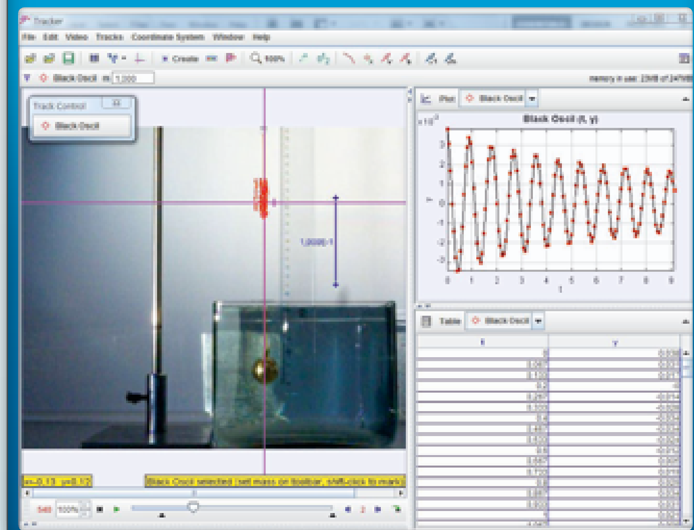
- The displacement values when velocity is maximum or zero
- The displacement values when acceleration is maximum or zero
- Why the period motion is two times the variation period of potential or kinetic energy
- The influence of friction on the parameters of the motion

CONCLUSION

The simple motion of a spring is not quite so simple to study. By both doing experimental work and working with real data for the chosen software, students will easily understand the dependence between the different parameters of oscillatory motion, and develop their ICT skills. They will be able to apply the acquired knowledge to the study of other oscillatory motions.



⑩ Testing a damped oscillation with a simple set of materials



⑪ Result of the analysis with Tracker