

Matrices and how they can be used in quantum computing – Quiz

Solutions

Question 1

The order of a matrix with 3 rows and 4 columns is 7.

A. True

B. False – The order of a matrix with 3 rows and 4 columns is 12.

Question 2

A square matrix is a matrix with equal numbers of rows and columns.

A. True

B. False

Question 3

Which operation requires matrices to have the same dimensions?

A. Multiplication

B. Addition

C. Transposition

D. Kronecker product

Question 4

In matrix multiplication, the number of columns in the first matrix must match the number of ____ in the second matrix.

A. Rows

B. Columns

C. Elements

D. Numbers

Exercise 1: Addition, subtraction and multiplication of matrices

For the given matrices A and B , calculate $A + B$, $A - B$, AB and BA .

$$A = \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

Solution:

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4+1 & 1+2 \\ 2+0 & -1+3 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}$$

$$\mathbf{A} - \mathbf{B} = \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4-1 & 1-2 \\ 2-0 & -1-3 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 2 & -4 \end{bmatrix}$$

$$\mathbf{AB} = \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 \cdot 1 + 1 \cdot 0 & 4 \cdot 2 + 1 \cdot 3 \\ 2 \cdot 1 - 1 \cdot 0 & 2 \cdot 2 - 1 \cdot 3 \end{bmatrix} = \begin{bmatrix} 4 & 11 \\ 2 & 1 \end{bmatrix}$$

$$\mathbf{BA} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 4 + 2 \cdot 2 & 1 \cdot 1 - 2 \cdot 1 \\ 0 \cdot 4 + 3 \cdot 2 & 0 \cdot 1 - 3 \cdot 1 \end{bmatrix} = \begin{bmatrix} 8 & -1 \\ 6 & -3 \end{bmatrix}$$

Note that $\mathbf{AB} \neq \mathbf{BA}$

Exercise 2: Kronecker product

A) Calculate the Kronecker product of the two vectors $\vec{u} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$.

Solution:

$$\vec{u} \otimes \vec{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \otimes \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \cdot 4 \\ 1 \cdot 2 \\ 3 \cdot 4 \\ 3 \cdot 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 12 \\ 6 \end{bmatrix}$$

B) Find the **quantum states** of a two-qubit system composed of the two qubits in the states $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Solution:

The **quantum states** of a two-qubit system composed of the two qubits in the states $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are:

$$|0\rangle \otimes |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = |00\rangle.$$

$$|0\rangle \otimes |1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = |01\rangle.$$

$$|1\rangle \otimes |0\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = |10\rangle.$$

$$|1\rangle \otimes |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = |11\rangle.$$

