

Part 3: The CNOT Gate and Entanglement

Exercises

Exercise 1. Check all of the examples listed above—where a gate is applied to one or two qubits—by creating the corresponding quantum circuits. See the short tutorials ‘How to access quantum hardware at IQM from Qiskit?’ and ‘How to access quantum hardware at IBM from Qiskit?’.

Exercise 2. Check all of the examples listed above—where a gate is applied to one or two qubits—in a local Qiskit environment. Enter your API token for the IBM Quantum system and run the programmes.

Exercise 3. Apply a NOT gate to the superposition states $\frac{1}{\sqrt{2}} \cdot |0\rangle + \frac{1}{\sqrt{2}} \cdot |1\rangle$ and $\frac{1}{\sqrt{2}} \cdot |0\rangle - \frac{1}{\sqrt{2}} \cdot |1\rangle$. You can either use matrix notation and bra-ket notation or assemble a quantum circuit or create Qiskit code. Describe the result in your own words.

Exercise 4. Take the example of a Hadamard gate being applied to a qubit. Explain why it is said that “a measurement reduces the quantum state of a system” (or: “the quantum state collapses”) and why multiple measurements are necessary to obtain a ‘reasonable’ result.

Exercise 5. Check whether the two qubits described by the state $\frac{1}{\sqrt{2}} \cdot (|11\rangle + |01\rangle)$ are entangled. What about two qubits represented by the state $\frac{1}{\sqrt{2}} \cdot (|01\rangle + |10\rangle)$? Use the method described in Part 3: 2. Entangling two qubits with a Hadamard and a CNOT gate.

Exercise 6. Use `print(circuit)` to print one of the quantum circuits from exercise 1, once before and once after the optimisation step. Describe what you notice. Are there quantum gates that you are not yet familiar with used in the optimisation step? Then research by which matrix they can be represented and what their effect is.

Exercise 7. Use the following code snippet to create and save the quantum circuit as an image (png, svg or LaTeX), once before optimisation and once after optimisation. Describe what you notice. Are there quantum gates that you are not yet familiar with used in the optimisation step? Then research by which matrix they can be represented and what their effect is.

```
from qiskit.visualization import circuit_drawer # for drawing circuits
```

```
circuit_image = circuit_drawer(circuit, output="mpl") # uses Matplotlib for creating the graphics; if LaTeX is installed, "latex" could also be used
```

```
circuit_image.savefig("Circuit (MPL).png") # save figure as pixel graphic (PNG)
```

```
circuit_image.savefig("Circuit (MPL).svg") # save figure as vector graphic (SVG)
```

Exercise 8. Verify the matrix calculations carried out in the lesson, i.e. for the Pauli-X gate, the identity gate, the Hadamard gate, the CNOT gate, and for the combination of Hadamard and CNOT gates. Compare the theoretical and the measured results – i.e. calculate the squares of the amplitudes representing and, thus, the probabilities of finding the qubit(s) in the different states. Compare these probabilities with the output(s) of the quantum computer.

Exercise 9 (optional). In addition to the Pauli-X gate, there is also a Pauli-Y gate and a Pauli-Z gate. Research their matrix representations and check their behaviour in a quantum circuit.

Exercise 10 (optional). It is possible to replace a quantum circuit by another equivalent quantum circuit (within reasonable approximations). A set of quantum gates that can replace *any* quantum circuit is called a *set of universal quantum gates*. Gather information on universal quantum gate sets. Compare quantum circuits that you have run before and after the optimisation. Which universal quantum gate set seems to have been used by the quantum computer?

Exercise 11 (optional). Look up various technical possibilities for physically implementing qubits in a quantum computer. Keywords: e.g. superconductors, ion traps.

Exercise 12 (optional). In 1996, David P. DiVincenzo introduced criteria for a quantum computer to work. One needs:

- well-defined qubits;
- qubits that are prepared in well-defined initial states;
- qubits that are stable for a sufficiently long period of time (decoherence time);
- a universal set of quantum gates;
- to be able to measure each qubit individually.

Try to explain each criterium using what you have learnt in this lesson. If you get stuck, do some research on the internet.